# ANALYSIS OF ECONOMIC CYCLES USING UNOBSERVED COMPONENTS MODELS

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**ABSTRACT**

The Unobserved Components Models represent a framework in which phenomena like any periodic behaviour, economic cycles in particular, may be modelled and forecast naturally. The main distinct feature of the methodology used in this paper is the use of a Dynamic Harmonic Regression model, characterised by time variable parameters that may vary following a rich family of models. This class of models are set up in a State Space context that takes advantage of the extraordinary flexibility of the recursive algorithms known as the Kalman Filter and Fixed Interval Smoother. Different versions of the models have to be applied to time series, depending on their time properties. In particular, time series with a non-constant period cycle has to be analysed in a totally different way to other series that exhibit a constant period cycle. A simple method to extract the cyclical information from the series in the case of non-constant period cycles is presented in the paper. The methodology is compared with others and shown working in practice with several examples.

***Key words:*** Hodrick-Prescott filter, Integrated Random Walk, Kalman Filter, Fixed Interval Smoother, Unobserved Components Models.

## 1. INTRODUCTION

It was long time ago when the necessity of extracting basic unobserved signals from observed time series appeared. One interesting aspect of such an special analysis of time series was the possibility to follow the underline evolution of variables independently of all kind of short run variations, such as seasonal patterns or outlier observations. In this regard, the study of trends, cycles and seasonal components (or alternatively seasonality-adjustment) have been the goals of many researchers worldwide.

Regarding the analysis of the economic cycles, a great deal of confusion has appeared in the literature since the initial studies about the topic. The important work of Burns and Mitchell (1946) "*Measuring the Business Cycles*" may be considered an initial step in the direction of the empirical evaluation of this matter. However, it was criticised by important researchers, like Koopmans (1947), mainly because it presented "measurement without theory". Nevertheless, Kydland and Prescott (1990) argued that while the criticism is acceptable, hypothesis about the statistical distributions of variables should not be done based on the Economic Theory.

These are examples of the widely accepted inherent subjectivity of this topic. While the distinction between trend and cycle is somewhat problematic from a theoretical point of view, however, the empirical definition is quite transparent if it is provided in the frequency domain and using Unobserved Components (UC) models.

When dealing with obtaining empirical measures of the economic cycle, the confusion is complex, because the lack of consensus about the definition of the economic cycle has motivated the appearance in the literature of a variety of very different methods. Some of the most typical are linear or polynomial trends; differentiation; application of moving average filters; Hodrick-Prescott (HP) filter; non-parametric estimates; etc. Due to the rather different nature of each of these procedures the estimated cyclical components for the same series have very different properties.

Burns and Mitchell made very important contributions to the problem that have been blurred by posterior discussions between theorists and practitioners. These basic contributions are the definition of the different phases of the cycle and its characteristics, but above all and from the point of view of the present paper, the explicit delimitation of the bandwidth of interest in order to provide a clear definition of the cycle (such bandwidth in their original work was between 18 and 120 to 144 months).

This paper presents a new UC methodology for the analysis of the economic cycle very successful in the past when applied to a wide range of time series. In addition, a general method for obtaining an estimation of the cycle based on a frequency domain approach is proposed. This method consists of the application of a band-pass filter to the series in which the bandwidth is defined by the user depending on his/her particular definition of the cycle. This filter is effectively built as a simple UC model within this new class applied twice to the series and is closely related to the HP filter.

## 2. THE DIVERSITY OF UNOBSERVED COMPONENTS MODELS

The UCM are models in which the time series are decomposed as the sum or a product of a number of other simple time series with economic or physical meaning. One widely accepted univariate version of UCM is

$$y_t = T_t + C_t + S_t + e_t \tag{1}$$

where $y_t$ is the observed time series; $T_t$ is a trend or low frequency component; $C_t$ is a sustained cyclical or quasi-cyclical component (e.g. an economic cycle) with period different from that of any seasonality in the data; $S_t$ is a seasonal component (e.g. annual seasonality); and $e_t$ is an 'irregular' component, normally defined for analytical convenience as a normally distributed Gaussian sequence with zero mean value and variance $s^2$. In order to allow for nonstationarity in the time series $y_t$, the various components in the model, including the trend $T_t$, can be characterised by stochastic, Time Variable Parameters (TVP's), with each TVP defined as a nonstationary stochastic variable, as discussed below.

While most researchers in the area would agree basically with the general formulation of the problem given by equation (1), the agreement ends just there. There are an endless amount of 'philosophical' and practical details in such a formulation that produces a big amount of alternative methodologies.

From all the methods available nowadays, one of the oldest and best known techniques, in essence better known as a seasonal adjustment method, is the **X-11** and extensions i.e. **X-11 ARIMA** and **X-12 ARIMA** (see e.g. Findley *et al*., 1996). Due to the fact that it has been the main method of trend estimation and seasonal adjustment used by Government Agencies all over the World, it has been applied mainly to macroeconomic monthly, quarterly and annual data. This is a rather *ad-hoc* method of seasonal adjustment in which smoothing procedures are used to extract trend and seasonal components from the time series. The design of these filters is based mainly on previous experience and they have been continuously refined as the method has been applied to more and more series. One of the most important limitations is its *ad-hoc* nature, forcing a continuos refinement as the method is applied to more time series.

Some deterministic optimisation methods have been proposed for signal extraction. These are considered from a variety of different standpoints such as **regularisation** (e.g., Hodrick and Prescott, 1980, 1997; Akaike, 1980) where constraints are imposed on the state estimates via a Lagrange Multiplier term within the cost function, in order to ensure that they possess the required characteristics. More specifically, the variance of residuals is usually minimised subject to a given degree of smoothness imposed on the components, as defined by specified weighting matrices in the Lagrange Multiplier term of the cost function. The values of Lagrange Multipliers are estimated in different ways, such as cross validation, but Akaike estimates them within a Bayesian framework. Other alternative standpoints for deterministic optimisation methods are 'smoothing splines'; 'smoothing kernels'; 'pseudosplines' and 'wavelet' methods.

There are two additional families of methods that have received a lot of attention, especially among economists. These are the *Structural form* and the *Reduced form* methods, following the analogy with the Simultaneous Equations models typical of Econometrics. If each of the components in equation (1) are assumed to follow ARIMA models (these are the structural form of the components), then the overall model, or reduced form, has to be another ARIMA of certain orders depending on the individual, structural, models. This two families have been viewed usually as competitors, given the radically different starting point of view.

In **Structural form** methods the UC model is considered as the observation equation of a discrete time stochastic State Space (SS) model and the associated state equations are used to model each of the components in Gauss-Markov form. This formulation has its origin in the the 1960's when control engineers realised that recursive estimation and, in particular, the *Kalman Filter*, could be applied to the problem of estimating time variable parameters in regression models, usually within a dynamic systems context. More recent developments have shown how this approach can be extended in various ways to problems of forecasting, backcasting, smoothing (by means of the recursive *Fixed Interval Smoothing*) and signal extraction.

Here, the most influential contributions are probably those of Harvey, whose *Structural Model* approach is now widely available in the successful STAMP computer programme (*Structural Time Series Analyser, Modeller and Predictor*: see Koopman *et al*, 1995; Harvey, 1989); and the Bayesian approach by Harrison and West, implemented in the *Bayesian Analysis of Time Series* programme (BATS: see Pole *et al*., 1995; West and Harrison, 1989).

The **Reduced Form** approach to UC model identification and estimation follows from the success of Box-Jenkins methods of time series analysis and forecasting (Box and Jenkins, 1970; Box *et al*., 1994). It is assumed that the series can be modelled as an ARIMA model (or reduced form) and find the structural models for the *n* individual components through a process of identification, i.e.

$$y_t = \frac{J(B)}{f(B)} e_t = \sum_{i=1}^{n} \frac{J_i(B)}{f_i(B)} e_{it}$$

where $B$ is the backward-shift operator; $J(B)$ and $f(B)$ are general MA and AR polynomials of order $q$ and $p$ identified and estimated *a la* Box-Jenkins[1]; each of the

[1] The term general is used here in the sense that MA and AR polynomials may, and in most cases will, be the result of a product of regular and seasonal polynomials. Also, the AR polynomial here contains all the unit roots in the model included by differencing in order that the process at hand may be treated as mean stationary.

$J_i(B)$ and $f_i(B)$ are unknown polynomials which orders would depend on the nature of each of the components[2] and the $q$ and $p$ orders of the overall ARIMA model.

From the previous formulae, it is obvious to see that there exist an infinite number of structural forms compatible with each reduced form. Therefore, an identification process is necessary in order to find a satisfactory solution to the problem. Such process relies on the imposition of a number of arbitrary constraints to ensure existence and uniqueness of the decomposition. Many different ways to achieve this goal have been proposed in the literature, being the most popular the 'canonical decomposition' by which the variance of the irregular component is maximised in order to obtain components as free from noise as possible (Hillmer *et al*., 1983). However, this is not the unique method, other less popular are e.g. the 'formal decomposition' and the 'statistical decomposition' (see Piccolo, 1982). A full methodology that takes advantage of the canonical decomposition is implemented in the software package known as *Signal Extraction in ARIMA Time Series* (SEATS, Gómez and Maravall, 1998).

The new methodology introduced in this paper falls into the Structural class of models, but with novel aspects discussed below, like new methods of estimation that takes advantage of the exceptional spectral properties of the Dynamic Harmonic Regression (DHR) model (see e.g. Ng and Young, 1990; Young *et al*., 1999). Such a methodology has been implemented in a software package, the multi-platform, CAPTAIN Time Series Analysis and Forecasting Toolbox in Matlab[3] (and its predecessor the MS-DOS based microCAPTAIN program).

## 3. The Structural Unobserved Components Time Series Model

In the Structural framework, (1) is considered as the observation equation in discrete-time (possible continuous-time of discrete differential) Non Minimal State Space model which describes the stochastic evolution of state variables associated with the UC's in (1). The SS form is an extraordinarily powerful and flexible tool for time series analysis, as it will be clear in the next pages. It is also a framework in which the UC type of models fits naturally.

The SS formulation is described by the following state and observation equations:

$$\begin{array}{lll} \text{State Equations} & : & \boldsymbol{x_t} = \boldsymbol{F}\boldsymbol{x_{t\text{-}1}} + \boldsymbol{h}_t \\ \text{Observation Equation} & : & y_t = \boldsymbol{H_t}\boldsymbol{x_t} + e_t \end{array}$$

[2] For example the trend model would incorporate the unit roots in the overall model; the seasonal component would include the seasonal roots; and so on.

[3] Information about this software is available in http://cres1.lancs.ac.uk/captain/ and a beta-test version is available from the author.

where $\boldsymbol{x_t}$ is an *n* dimensional state vector; $y_t$ is the scalar observed variable; $\boldsymbol{h}_t$ is an *n* dimensional vector of zero mean, white noise inputs (system disturbances) with diagonal covariance matrix $\boldsymbol{Q}$ and $e_t$ is the white noise variable in equation (1), which is assumed to be independent of $\boldsymbol{h}_t$. $\boldsymbol{F}$ and $\boldsymbol{H_t}$ are, respectively, the *nxn* state transition matrix and the *1xn* observation vector, which relates the state vector $\boldsymbol{x_t}$ to the scalar observation $y_t$.

Given this form for the overall model, the well-known Kalman Filter and the associate recursive Fixed Interval Smoothing algorithm provide the basis for forecasting, interpolating and smoothing (that is, estimate the components). For a data set of *N* samples, the former algorithm yields a 'filtered' estimate of the state vector at every sample *t*, based on the time series data up to sample *t*. The later produces a 'smoothed' estimate of the states which, at every sample *t*, is based on all *N* samples of the data. This means that, as more information is used in the later estimate, its Mean Square Error cannot be greater than the former. As these algorithms are discussed in detail in the previous references, we will not pursue the topic further.

These algorithms allow inherently for missing data and provide automatic forecasting, interpolation and backcasting. The off-line FIS algorithm is particularly useful for interpolation, signal extraction, seasonal adjustment and lag-free TVP estimation. Also it is useful in the so call 'variance intervention' for handling sudden changes in the trend level.

Previous to the application of these algorithms, variances of all the noises $\boldsymbol{h}_t$ and $e_t$ (often called *hyper-parameters*) present in the model must be known or estimated in some way. A usual way to deal with the problem is to formulate it in Maximum Likelihood (ML) terms (e.g. Harvey, 1989). Assuming that all the disturbances in the state space form are normally distributed, the ML function can be computed using the Kalman Filter via 'prediction error decomposition'. This is the generally accepted method, because of his well-known theoretical basis. However, the optimisation can be very complex even for relative simple models due to the flatness around the optimum (Young *et al*., 1999).

There are several alternatives to ML, from which the one preferred here is built in the frequency domain (Young *et al*., 1999). Basically, the parameters are estimated so that the logarithm of the model spectrum fits the logarithm of the empirical pseudo-spectrum (either an AR-spectrum or periodogram) in a least squares sense. Such method provides the estimation of a 'noise variance ratio' matrix (*NVR*), instead of the variances in the SS model. This *NVR* matrix is defined as the ratio of the state noise covariance matrix to the variance of the observation equation noise i.e. $\underline{\boldsymbol{NVR}} = \boldsymbol{Q} / s^2$. Since $\boldsymbol{Q}$ is assumed diagonal, it is common to speak about the *NVR* parameters, referring to the diagonal elements of the $\underline{\boldsymbol{NVR}}$ matrix.

In the UC context all the components in (1) has to be represented in SS form, and then an overall SS form for the whole model is built by the assembly of the individual models.

Very often, the Generalised Random Walk (GRW) is the preferred model for the trend, given by

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_t = \begin{pmatrix} a & b \\ 0 & g \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{t-1} + \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}_t \qquad T_t = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_t \tag{2}$$

Here, $a$, $b$, and $g$ are constant parameters; $T_t$ is the trend component (i.e. the first state, $x_{1t}$); $x_{2t}$ is a second state variable (generally known as the 'slope'); $h_{1t}$ and $h_{2t}$ are zero mean, serially uncorrelated white noises with constant block diagonal covariance matrix $\boldsymbol{Q}$.

This model includes, as special cases, the Random Walk (RW: $a = 1$; $b = g = 0$; $h_{2t} = 0$); Smoothed Random Walk ($0 < a < 1$; $b = g = 1$; $h_{1t} = 0$); the Integrated Random Walk (IRW: $a = b = g = 1$; $h_{1t} = 0$); the Local Linear Trend (LLT: $a = b = g = 1$); and the Damped Trend ($a = b = 1$; $0 < g < 1$). In the case of the IRW, $x_{1t}$ and $x_{2t}$ can be interpreted as level and slope variables associated with the variations of the trend, with the random disturbance entering only via the $x_{2t}$ equation.

With respect to periodic components (either cyclical or seasonal) there are many different alternatives quoted in the literature. The main ones are the Dummy Seasonality; the Trigonometric Cycle or Seasonality (West and Harrison, 1989; Harvey, 1989); the Dynamic Harmonic Regression (DHR: Ng and Young, 1990; Young *et al.*, 1999); the Modulated periodic components (Young *et al.*, 1999); and the General Transfer Function model (Ng and Young, 1990).

From all the above, the DHR option is the one prefered in this paper. The DHR model has a linear regression form with deterministic periodic functions of time as inputs with TVP that follow GRW models. The SS form for this option is

$$\begin{pmatrix} a_f \\ a^*_f \end{pmatrix}_t = \begin{pmatrix} a_f & b_f \\ 0 & g_f \end{pmatrix} \begin{pmatrix} a_f \\ a^*_f \end{pmatrix}_{t-1} + \begin{pmatrix} h_f \\ h^*_f \end{pmatrix}_t$$

$$S_t = \sum_{f=1}^{R} \left\{ a_{ft} \cos(w_f t) + b_{ft} \sin(w_f t) \right\}$$

where $b_{ft}$ are parameters defined in the same way as $a_{ft}$. The rigidity introduced by the deterministic functions is compensated by the TVP giving the model a great flexibility, capable of represent many types of different seasonal patterns. This is extremely useful for series which exhibit the kind of non-stationarity behaviour in the seasonal component so commonly observed in economic time series.

## 4. THE ANALYSIS OF CYCLES USING STRUCTURAL UCM

Two particular versions of the models shown above are especially relevant for the research of cycles in time series. These are (i) a single IRW trend and (ii) a model that includes an IRW trend and a DHR model for the cyclical and/or seasonal part. The selection between these two models and the advantages of each of them depend on the properties of the time series and the objectives of the analysis, as illustrated below.

It is obvious that the objective of producing forecasts of the time series (or the cyclical component) into the future is much more complicated than analysing the peaks and troughs of the economic cycle into the past. For the present author, analysing and forecasting are two distinct objectives that may be treated separately in this context in the sense that a valid model for the analysis of the periodicities of the data in the past do not necessarily has to be a good model for forecasting the series into the future. However, a valid model in forecasting terms may be also valid for the analysis of the economic cycle inside the estimation sample. In addition to this, forecasting is also considerably more complicated for series that exhibit a non-constant period cycle, than for those series with a constant period cycle, and models used should reflect these features of the data.

Based on the previous considerations, the rest of this section is devoted to the analysis of two different scenarios, illustrated with time series that show the power of the UC models. The first scenario is a time series with a constant period for which a model may be built and it, in general, will be useful both for the 'structural analysis' and for forecasting objectives. The second scenario is one in which the series is somewhat more complicated, because the period of the cycle is time varying, in this case the 'structural analysis' of the cycle is relatively simple, but forecasting is far more complicated.

### 4.1. Cyclical components with a constant period.

The monthly aggregated electricity consumption in Spain from January 1971 to April 2000 in KW per hour is shown in figure 1 and is used here in order to exemplify the results obtained with a number of methodologies. These models are DHR; the Basic Structural Model (BSM) of Harvey; and an ARIMA model (in which the estimation of components is carried out using the canonical decomposition given in Gómez and Maravall, 1998).

The series reveals the nonstationarity behaviour in the mean and variance, the amplitude of which seems more important in the middle of the series. Pre-processing of the data in BSM and ARIMA methodologies are compulsory in these circumstances. In order to implement ML estimation in the BSM model, the logarithmical transformation was done. Apart from this, a regular and seasonal difference has to be done previous to the application of the ARIMA methodology. However, DHR models are able to avoid any pre-judgement of this kind by modelling and forecasting the basic data without any transform at all. In this way the DHR model explains the nonstationarity in both the mean and the variance of the original data, instead of removing such problems. However, in order to facilitate comparisons among the methods considered, the series is also logarithmically transformed for DHR modelling.

(INSERT FIGURE 1)

The standard identification tools suggest an ARIMA 'airline' model for the logarithmically transformed series (i.e. $\text{ARIMA}(0,1,1)\times(0,1,1)_{12}$). On the other hand, the identification in the frequency domain, necessary for the DHR analysis, reveals the existence of the trend and a clear seasonal pattern in the series confirmed by the AR spectral estimates, with visually significant peaks at periods of 12, 6, 4, 3, 2.4 and 2 months.

DHR models were estimated in the frequency domain using the CAPTAIN Matlab version (Young *et al*., 1999); BSM models were estimated by ML implemented in STAMP (Koopman *et al*, 1995); and ARIMA models were estimated by Exact ML using SEATS (Gómez and Maravall, 1998). Table 1 exhibits some relevant comparisons among the innovations obtained by the models. In particular, it shows the estimated models for the logarithmically transformed data in terms of the variance of the innovations; the Ljung-Box autocorrelation test for 12 and 24 lags; and the Jarque-Bera normality test[4].

The immediate conclusion extracted from table 1 is that the DHR model outperforms in statistical sense the other two methodologies. This is specially interesting, because the DHR model is optimised in the frequency domain, while BSM and ARIMA are both estimated in the time domain. It is surprising in the sense that ML criterion is defined as an explicit, time domain function of the normalised innovations, whereas optimisation in the frequency domain, is primarily concerned with ensuring that the spectral properties of the estimated components match the empirical spectrum of the data.

| | **DHR Model** | **Basic Structural Model** | **ARIMA** |
|---|---|---|---|
| $\hat{s}_a^2$ | 0.58e-03 | 0.64e-03 | 0.86e-03 |
| **Q(12)** | 13.98 | 16.89 | 14.24 |
| **Q(24)** | 22.79 | 29.69 | 25.46 |
| **Jarque-Bera** | 0.06<br>(0.96) | 1.14<br>(0.56) | 0.53<br>(0.76) |

**Table 1:** Innovations variance, Ljung-Box and Jarque-Bera normality tests of some models for the electricity consumption series in Spain.

Due to convergence problems arising in ML estimation for BSM models when individual hyper-parameters are estimated for each harmonic in the seasonal component, it is a common practice to constrain all of them to a single parameter. It is the case of the STAMP and other similar computer programs (see e.g. Koopman *et al.*, 1995; Pole *et al*, 1995). This may be one reason explaining the superiority of the DHR in frequency domain seen in table 1. The DHR model effectively has a higher number of parameters. Nevertheless, the computational burden of such estimation is less important than in time domain for BSM and ARIMA models, provided the spectral peaks are well defined, as it is the case of most seasonal economic data. There are also advantages in forecasting terms, but these are discussed later[5].

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[4] These statistics are estimated consistently by the author based on the innovations extracted from the respective software packages, instead of using the statistical output of each package, in order to ensure the comparability of results.

[5] Results similar to the ones presented here can be found in Young *et al*. (1999) for the well-known airline passengers time series in the US in Box and Jenkins (1970).

The signal extraction exercise reveals rather nicely another feature in the series that is hardly distinguishable in the original raw data; namely the fact that the trend appears to include a long period cycle of just over ten years which is, presumably, related to the economic cycle (see the estimated trend in figure 1). Such cyclical behaviour is also clear by means of the trend derivatives, a variable that has been used commonly for the analysis of the so called "underlined growth". Figure 2 shows the derivatives of IRW, LLT and ARIMA trends obtained by the three methodologies discussed above. It seems clear that the trends other than IRW are very dangerous in the analysis of the underlined growth because of the high level of noise. On the other hand, the smoothness of the IRW trend supports the existence of an approximately constant period cycle of about four years in the AP series.

(INSERT FIGURE 2)

The cyclical component detected may be included into the model in several ways, but the option chosen here is the one already exploited in Young *et al*. (1999). It consists of obtaining a better estimate of the spectrum at the low frequencies relating to the cycle by means of a higher order AR spectrum. An AR(120) spectrum shows a clearly defined peak in the spectrum at a fundamental period of about 126 months, with two other peaks at 60 and 37 months, which will be recognised as approximate harmonics of this fundamental period. Simply by concatenating the original AR(18) and the new AR(120) spectra, using the higher order AR spectrum to define the lower frequency cyclical band of the spectrum, and the lower order AR spectrum to specify the higher frequency seasonal behaviour, the model including the cycle can be estimated in the frequency domain. The resulting AR-spectrum is shown in figure 3.

(INSERT FIGURE 3)

In order to compare the forecasting results, four models are used. These are the three models shown in table 1 and a DHR model that incorporates periodic components at the frequencies of the long term cycle.

A rolling experiment was performed in order to make forecasting comparisons. The last six years of data were reserved for the comparative study. The four models were estimated and used to obtain up to 24 step-ahead forecasts. The process is repeated, expanding the sample by one observation at each step. The results of this exercise are shown in figure 4, where the mean, minimum, maximum and standard deviations of the Mean Absolute Percentage Error (MAPE) are plotted as a function of the forecast lead time. The thick solid lines show the results for the DHR model including the economic cycle; the thin solid line represent the simpler DHR model; the dotted lines represent the results for the BSM model; and the dashed lines show the ARIMA results.

(INSERT FIGURE 4)

It is clear that the two DHR model forecasts, with and without the cyclical component, are both significantly better than those obtained using BSM and ARIMA over the range of forecast lead times. Moreover, the superiority of the DHR model results increases substantially as the forecast horizon increases, especially in the case of the improved DHR model.

### 4.2. Cyclical components with a non-constant period.

All the models discussed in the previous subsection are set up for time series which exhibit a constant period cycle. It is obvious that those models are strictly incorrect when such assumption is not fulfilled and the specification error would be more serious when the cyclical component is relatively more important with respect to the rest of components.

It seems surprising that the type of models in the previous sections have been used by qualified analysts for time series which irregularity of the periodic behaviour is widely acknowledged. From the point of view of the current authors, such analysis may be dangerous, especially when used for forecasting purposes, but not necessarily for the structural analysis of the cycles.

Typical examples of the above problem are the US National Accounts quarterly time series, that have been used systematically for the analysis of the economic cycle from very different theoretical and practical points of view, both for the analysis of the economic cycle itself or for the detection of co-movements among variables coherent with some theoretical dynamic macroeconomic models. In a significant amount of these studies, the Hodrick-Prescott filter (HP; see Hodrick and Prescott, 1980, 1997) plays a central role as a mean to obtain the periodic components of the series.

Two most interesting procedures for the estimation of cyclical components for the US GNP series taken from the literature are:

a) Definition of the economic cycle as the perturbation around a HP trend, that is effectively a rather simple UCM (see below).

b) UCM that includes explicitly a cyclical component, in addition to the trend (Koopman *et al*., 1995).

Although option b) seems a more comprehensive model because of the specific inclusion of the cycle in the model, the cyclical component is effectively estimated once more as the difference between the series and a HP trend. This is a consequence of the blind Maximum Likelihood estimation process in which the period of the cyclical component and the rest of hyper-parameters are estimated jointly. To be exact, when Harvey's BSM was estimated for the logarithmically transformed US GNP series (from the second quarter of 1948 to the second quarter of 1998) the optimum was found with 'very strong convergence' at the following solution

$$y_t = T_t + C_t + e_t$$

$$\text{Irregular:} \quad \sigma_e^2 = 0$$

$$\text{Trend:} \quad \sigma_{Level}^2 = 0 \qquad \sigma_{Slope}^2 = 0.0012$$

$$\text{Cycle:} \quad \sigma_{Cycle}^2 = 0.008 \qquad \rho_{Cycle} = 0.902 \qquad \text{Period} = 4.37 \text{ years}$$

In words, this model implies that the trend is effectively a HP type (but with a different smoothing constant than the HP filter, see below) with no irregular component and with a cycle that has a period somewhat longer than four years. In practice, this model is rather similar to the previous one, in which we were assuming that the series is just a HP trend plus a perturbational component about it, with the only difference that the smoothing constant for the HP filter is different. One may wonder whether the BSM model above is a sensible option or not in general, mainly because the model is initially specified including an irregular component, but the output is a model without it, meaning that a model has no stochastic noise added to it.

The proposal of this paper is a mixture of the two models seen above , in which the cycle and an irregular components are explicitly introduced in the model. At the same time, the non-constant features of the cycle are directly addressed by means of the exact definition of the cycle in terms of a frequency band.

The HP trend is set up in a regularisation framework in which smooth signal, the trend, is extracted from the data in a way such that the sum of squares of residuals are minimised subject to the constraint that the trend must have a certain level of smoothness. Such smoothness is controlled by a lagrange multiplier that is maintained at a value of 1600 for quarterly series. It has been demonstrated (e.g. see Pedregal, 1995 and Young and Pedregal, 1996) that the HP filter is exactly equivalent to a simple UC model composed of an IRW trend plus an irregular component. In addition, it has been shown that the IRW filter is effectively a low pass filter in which the bandwidth is controlled by the NVR parameter, that is exactly the inverse of the smoothing constant in the HP filter. In this way, the filtered series would contain all the information of the series in the defined frequency band, independently of the systematic or asystematic properties of the series. The exact formulae is

$$y_t = T_t + e_t \qquad T_t = \frac{h_t}{\nabla^2} \qquad NVR = \frac{s_h^2}{s_e^2} \tag{3}$$

Pedregal (1995) gives the exact formulae to calculate the bandwidth of the RW family filters as a function of the NVR parameter. Table 2 shows some examples for an IRW filter.

| NVR | Period (Time samples) | Years (quarterly series) | Years (monthly series) |
|---|---|---|---|
| **10** | 2.86 | 0.71 | 0.24 |
| **1** | 6.01 | 1.51 | 0.51 |
| **0.1** | 11.02 | 2.76 | 0.92 |
| **0.01** | 19.78 | 4.95 | 1.65 |
| **0.001** | 35.28 | 8.82 | 2.94 |
| **1/1600** | 39.69 | 9.92 | 3.31 |
| **0.0001** | 62.81 | 15.71 | 5.23 |

**Table 2:** Relation between the *NVR* parameter and the associated bandwidth of the IRW filter, i.e. minimum period of cycles include in filtered series.

Based on table 2, if a filtered series has to be estimated in a way that contains all the information of the time series about 5 years and above, a *NVR*=0.01 would be the option for a quarterly series, while a *NVR*=0.0001 should be chosen for a monthly series.

Now, if an exact definition of the cycle is provided in terms of a frequency band, an estimation of such cycle may be obtained by the application of the IRW twice, in order to build a band pass filter. In the first pass a trend-cycle component would be obtained by applying the IRW filter with an appropriate *NVR*. In the second pass, the trend-cycle (or the original series) is filtered again in order to remove the low frequency information related to the trend and to get the cyclical information of the series. The result of such a process is a decomposition of the series in a low frequency trend; a cycle with constant or non-constant period depending on the properties of the series; and an irregular component that will not be white noise necessarily.

For example, applying Burns and Mitchell (1946) definition of the cycle to the US GNP quarterly series, such component would contain the information of the series between 18 months (6 quarters) and 120 to 144 months (40 to 48 quarters). The proposed estimation of the cyclical component could be obtained in two steps, i.e.

(1) Apply model (3) with *NVR*= 1. The filtered series would contain all the information of the series about 6 quarters and above (trend-cycle component).

(2) Apply model (3) to the trend-cycle component in (1) or to the original series with *NVR*= $2.92 * 10^{-4}$. The filtered series would contain the information about 48 quarters (12 years) and above.

The estimated cycle is the perturbation in (2), if (2) is applied to the trend-cycle component in (1), or the difference between the trends in (1) and (2), if (2) is applied to the original time series. The actual differences between applying (2) to the original series or to the trend-cycle in (1) are negligible.

This method exhibits a number of advantages:

- It relies on an explicit definition of the cycle based on a frequency band, instead of defining it as a residual (HP) or just one frequency or a narrow band around it (BSM). This may be seen as a matter of taste, and it is indeed so for the HP approach, but may be dangerous the definition of the cycle given by BSM for series with a non-constant period cycle.

- It does not require any preprocessing of the data (e.g. variance stabilisation) that would be necessary in the BSM approach.

- Exactly the same procedure, as it is without any addition, may be applied to seasonal data. The BSM model may be applied to such data with the (simple) addition of a seasonal component, but the cyclical component estimated by HP for seasonal data would incorporate the seasonal component, and the correction of this problem do not seem easy within the HP context at first sight.

- An irregular component is always estimated from the data, avoiding this rare peculiarity of some UC methodologies.

(INSERT FIGURE 5)

Figure 5 shows the application of the three alternatives (BSM, HP and IRW) mentioned in this subsection to the GNP series. It is clear from the plot that all are very similar, though the theoretical definition is quite different in each model: the BSM model produces a cycle that is effectively the information of the series of period about 3.47 years; the HP filter assumes that the cyclical component incorporates the information of period less than 9.9 years (see table 2); while the IRW cyclical component is defined as the information of the series between 1.5 and 12 years. The logical consequence is that most of the information of this series is precisely contained in the band between 1.5 and 10 years (approximately), while the rest of the frequencies are much less relevant. Because of this, differences among estimated cycles are expected if we play within this frequency band.

The problem of figure 5 is the lack of smoothness that may induce to a number of oscillations superior to what one would like for the analysis of the cycle: some of them are indeed very short, some of them of very low amplitude, making difficult the detection of recessions and expansions. One possibility to solve such a problem is selecting a different bandwidth for the cycle components. For example, figure 6 shows some alternatives for different bandwidths, namely between 1.5 and 12 years (as in figure 5); 4 to 12 years and 6 to 12 years.

The final choice of the frequency band is subjective because the conclusions about the cyclical oscillations are different. For example, given the definitions of the cycles in figure 6, the smoother estimate cannot see the cycles of period inferior to six years. This is the reason why this estimate considers one single cycle between years 1950 and 1958; 1965 and 1971; 1984 and 1993; while the other estimates would consider the existence of two cycles within each of those dates. On the other hand, the least smooth cycle estimate consider a very short cycle between 1959 and 1961. In order to define a sensible bandwidth, other kind of analysis, typically recessions and expansions records made by experts in the field, become very relevant.

(INSERT FIGURE 6)

## 5. CONCLUSIONS

This paper proposes new alternatives for the analysis of the economic cycles related to well-known generally available methods, but with a number of novel features. The analysis of the cyclical behaviour of time series fits naturally in the Unobserved Components (UC) models framework. From all the complex diversity of such methods, the one proposed in the main text is of the Structural form type. Two versions are proposed, based on the specific properties of the data at hand.

Firstly, a Dynamic Harmonic Regression for those series that exhibit a constant period cycle. The relevance of such model is demonstrated with the monthly aggregated electricity consumption data in Spain. In this series, the forecasting performance of a model that includes a cycle (estimated objectively from the data) is considerable superior to other simpler univariate UC and ARIMA models.

Secondly, a double process of filtering for series with non-constant period cycles is proposed. It is based on a very simple UC model (just an Integrated Random Walk trend) that provides the estimation of the information content of the series within a frequency band that should be provided subjectively by the user. The results of such procedure compare very favourably with the Basic Structural Model and Hodrick-Prescott trend modelling.

## 6. REFERENCES


Akaike, H. (1980). Seasonal adjustment by a bayesian modelling. Journal of Time Series Analysis, 1, pp. 1-13.

Box, G. E. P., and Jenkins, G. M., (1970, 1976). Time Series Analysis, Forecasting and Control. Holden-Day: San Francisco.

Box, G.E.P., Jenkins, G.M., Reinsel, G.C. (1994). Time Series Analysis, Forecasting and Control. Prentice Hall International, Englewood Cliffs, New Jersey.

Burns, A.M. and Mitchell, W.C. (1946). Measuring Business Cycles. NBER, New York.

Findley, D. F., Monsell, B. C., Bell, W. R., Otto, M. C. and Chen, B.C. (1996). New capabilities and methods of the X-12 ARIMA seasonal adjustment program. U.S. Bureau of the Census. Mimeo, May 16.

Gómez, V. and Maravall, A. (1998). Programs TRAMO and SEATS, Instructions for the User (Beta Version: June 1998). Bank of Spain, Madrid.

Harvey, A. C. (1989). Forecasting Structural Time Series Models and the Kalman Filter. Cambridge University Press, Cambridge.

Hillmer, S. C., Bell, W. R. and Tiao, G. C. (1983). Modelling Considerations in the Seasonal Adjustment of Economic Time Series. In Zellner, A. (ed), Applied Time Series Analysis of Economic Data, US Dept. of Commerce-Bureau of the Census, Washington D. C., pp. 74-100.

Hodrick, T. and Prescott, E. (1980). Post-war US business cycles: an empirical investigation. Carnegie Mellon University, manuscript. (1997). Journal of Money, Credit and Banking, 29, pp. 1-16.

Koopman, S. J., A. C. Harvey, J. A. Doornik y Shephard, N. (1995). STAMP 5.0: Structural Time Series Analyser, Modeller and Predictor. Chapman & Hall, London.

Koopmans, T. (1947). Measurement without theory. Review of Economic Statistics, 29, pp. 161-172.

Kydland, F.E. and Prescott, E.C. (1990). Business cycles: real facts and a monetary mith. Federal Reserve Bank of Minneapolis Quarterly Review, Spring, pp. 3-18.

Ng, C. N. and Young, P. C. (1990). Recursive Estimation and Forecasting of Nonstationary Time Series. Journal of Forecasting, 9, pp. 173-204.

Pedregal, D.J. (1995). Comparación Teórica, Estructural y Predictiva de Modelos de Componentes no Observables y Extensiones del Modelo de Young. PhD thesis. Universidad Autónoma, Madrid.

Piccolo, D. (1982), 'A Comparison of Some Alternative Decomposition Methods for ARMA Models', in O. D. Anderson (ed), *T*ime Series Analysis: Theory and Practice 1, North-Holland Publishing Company.

Pole, A., West, M. y Harrison, J. (1995). Applied Bayesian Forecasting and Time Series Analysis. Chapman & Hall, New York.

West, M. and Harrison, J. (1989). Bayesian Forecasting and Dynamic Models. Springer-Verlag, New York.

Young, P.C. and Pedregal, D.J. (1999). Recursive and En-Block Approaches to Signal Extraction. Journal of Applied Statistics, 26, pp. 103-128.

Young, P.C., Pedregal, D.J. and Tych, W. (1999). Dynamic Harmonic Regression. Journal of Forecasting, 18, pp. 369-394 .

## FIGURES

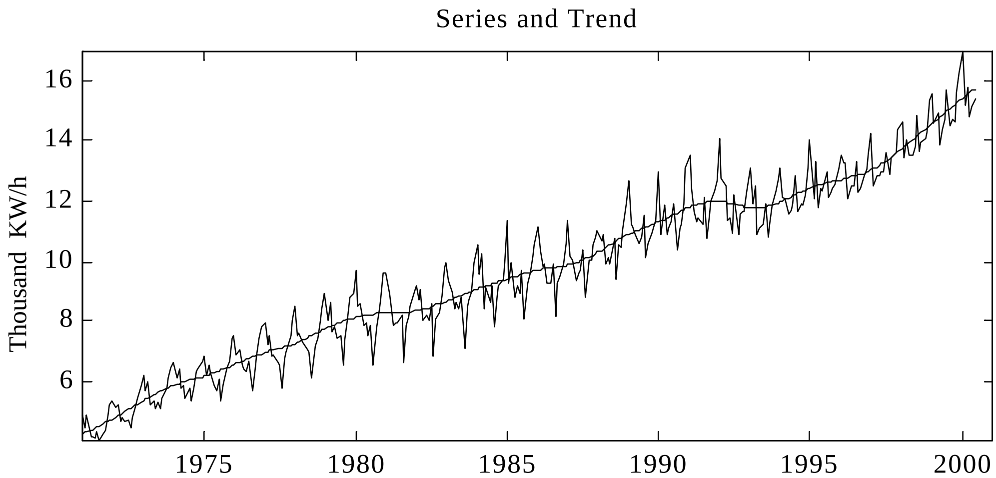


**Figure 1:** The Electricity consumption in Spain from January 1971 to April 2000, and estimated trend.

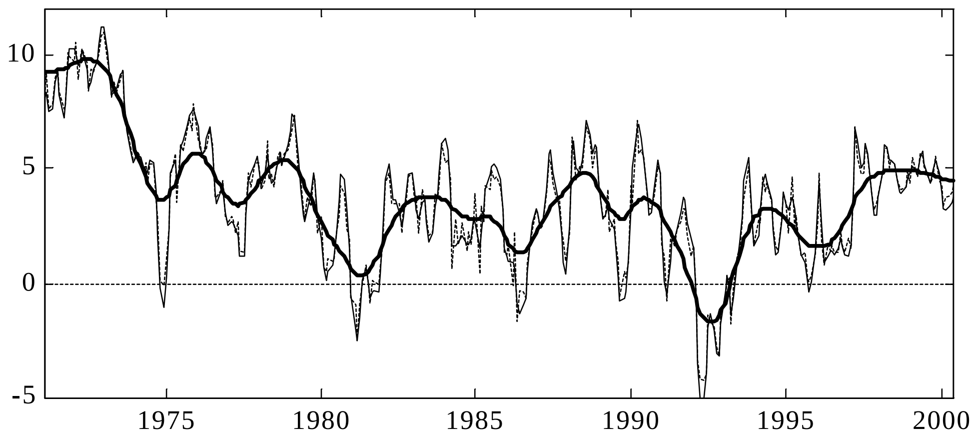


**Figure 2:** Trend derivatives of alternative models: IRW trend in DHR model (wide solid); canonical decomposition trend in ARIMA model (thin solid); and LLT in BSM model (dotted).

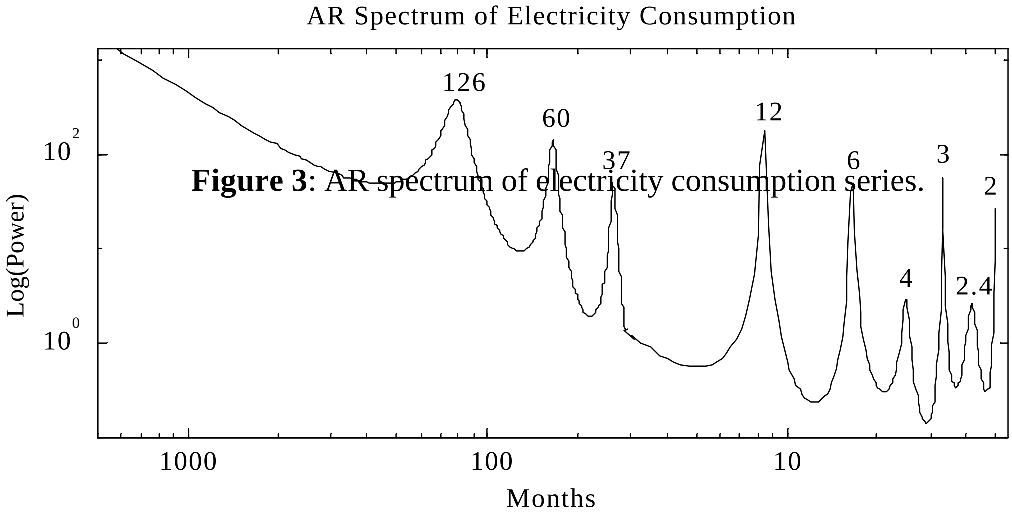


**Figure 3**: AR spectrum of electricity consumption series.

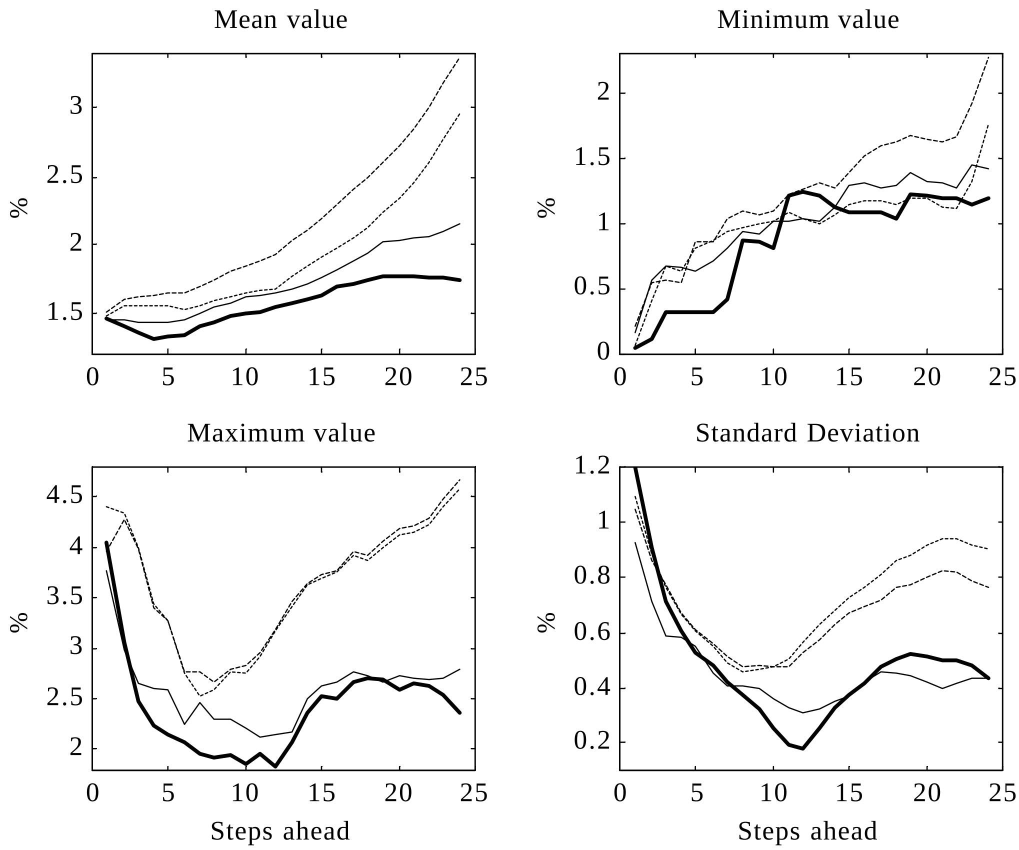


**Figure 4:** Relative forecasting performance for the electricity consumption series in Spain, based on MAPE measures. DHR seasonal plus cyclical model results (thick

solid); standard seasonal DHR model results (thin solid); BSM results (dotted); and ARIMA results (dashed).

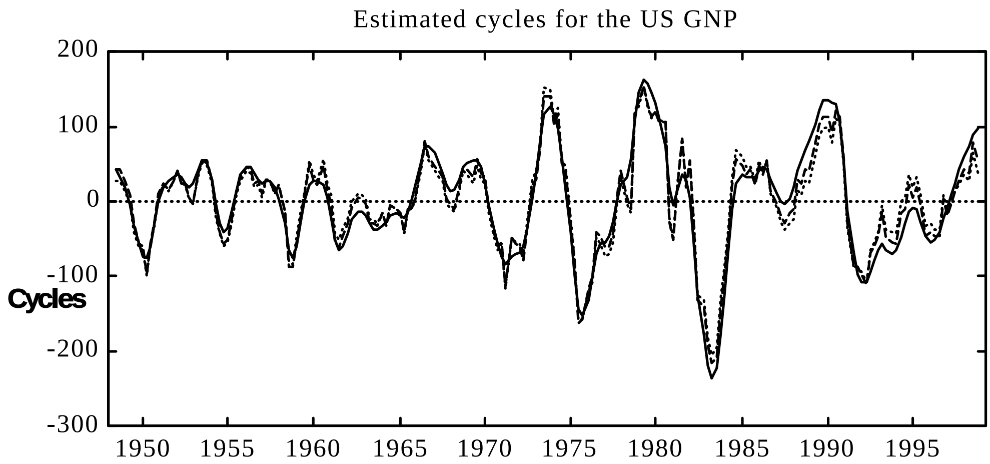


**Figure 5:** Three possible estimations of the cyclical component for the US GNP series: IRW filter (solid); HP (dashed) and BSM (dotted).

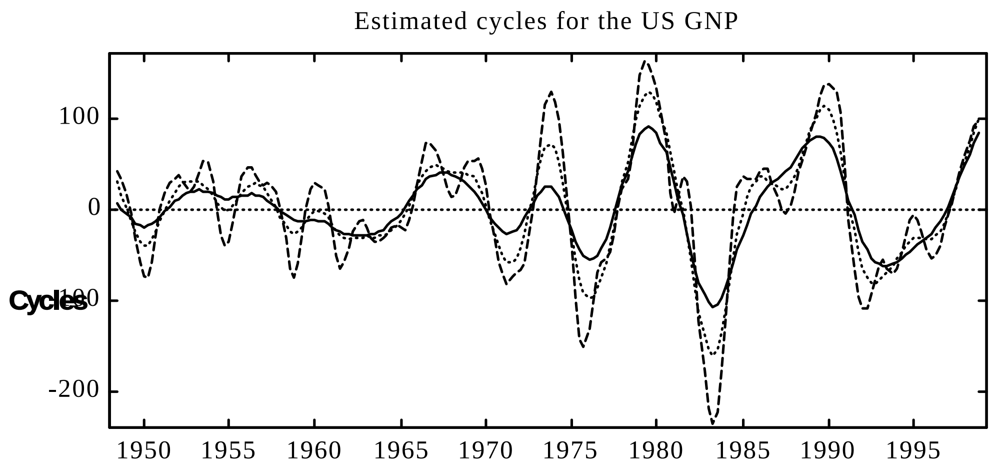


**Figure 6:** Estimated cycles for the US GNP using the IRW approach for different bandwidths: from 1.5 years to 12 years (dashed); 4 to 12 years (dotted) and 6 to 12 years (solid).