More on the Short Cycles of Interest Rates

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In an article published earlier in this journal [4], we studied the term structure of interest rates in a dynamic context. Instead of focusing on the yield curve at a point in time, we investigated the joint movement of short and long-term interest rates through time. We compared the cyclical behavior of the ninety-day Treasury bill rate and the ten-year U.S. government bond rate by using cross-spectral analysis. The data used for the analysis were obtained from regression-fitted yield curves. These fitted yield curves enabled us to obtain the monthly yields of securities of prespecified term to maturity. The derivation was done in a precise manner which at the same time is in line with most of the previous term structure studies.[1]

Our results show that a short-run cycle of eighteen to twenty-four months is significant in both series for the period 1954-1967. Over the cycle, the long-term rate leads the short-term rate and, as the cycle length increases, the relative lead period becomes shorter. The finding of a subcycle (which is shorter than the business cycle) is in line with what was found by other researchers with respect to other economic variables.[2] The findings that the long rate leads the short rate are consistent with the expectations hypothesis of the term structure of interest rates.[3]

In his comment on our article, Dr. Percival argues that the results are questionable because we do not use statistical inference tests in our analysis

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[1] See Melnik and Kraus [4, pp. 293-294].

[2] See for example Ruth P. Mack [3]. Mack found that a number of economic time series contain cyclical components which are shorter than that of business cycle. The average duration of these subcycles is twenty-four months and some of them are even six months shorter. These sybcycles are on the average about half the length of the regular business cycle. See also Granger and Hatanaka. [1, pp. 16-18 and 263-270].

[3] The finding of a lead of the long rate over the short rate is confirmed by Sargent [5]. In footnote 4 of our article [4], we tried to explain the economic reasons for the statistical result.

and because we use deviations from a fitted trend line to correct for the nonstationary nature of the rate series (rather than the method of using first differences). Dr. Percival then presents cross-spectral results of his own study to refute our conclusion. His data, however, are completely different in nature from ours and, therefore, his results as stated do not contradict ours.[4]

Dr. Percival's comment, we feel, is important not so much because of the "interest-rate cycle" issue but rather because it focuses on two possible problems in the interpretation of spectral analysis in general: the definition of "significant" cycles and the method of correcting for nonstationarity.[5] Therefore, we proceed to discuss these problems and to discuss their implications to our interest-rate analysis.

The spectral approach extends the traditional decomposition of time series into trend, cyclical, seasonal, and random movements. It is based on a somewhat rigorous method which decomposes stationary time series into many uncorrelated components each of which is associated with a period or a frequency.[6] A stationary series might be viewed as a series which is in a statistical equilibrium, in the sense that it contains no trends.[7] Economic time series are for the most part nonstationary, and therefore most studies dealing with them use techniques for removing or filtering out the nonstationary part, leaving behind a series that can be treated as stationary.

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[4] The major difference in the data is that our interest-rate data belong to securities with constant term to maturity (i.e., taken from a yield curve) while Percival's data are taken as they are from the *Federal Reserve Bulletin*. The long-term rate in his analysis is not specifically a ten-year rate or, for that matter, of any prespecified maturity. The term to maturity of the securities whose rates are quoted in the bulletin changes from month to month. For the difference between such data and the data that we use, see Melnik and Kraus [4,p.293]

[5] The procedure that Dr. Percival recommends in his note for analyzing cross-spectral results is not new and is not really a criticism of our result. We fully agree with him that cross-spectral analysis is called for only if common cycles appear to exist in the auto-spectral curves. We did not spell out the results of the auto-spectral analysis because we wanted explicitly to discuss the *relationship* of short-run and long-run interest rates. Cross-spectral analysis is appropriate for our declared purpose because it indicates the degree of linear association, timing relationship, and amplitude of our two time series.

[6] See Ransser and Cargill [6] and Jenkins and Watts [2].

[7] One consequence of assuming a stationary process is that the joint probability function $f_{12}(x_1,x_2)$ depends only on the time difference and not on the absolute values of t_1 and t_2. The stationarity concept refers to statistical equilibrium in both the mean and the variance.

A particularly simple form of removing the trend (or low-frequency components in general) is to use the first deference of the original series $y_t=(x_t-x_{t-1})$ where x_t is the original series. Another possible method is to assume that the basic series is of the form $x_t=y_t-m(t)$ where y_t is a stationary series. In this case removing m(t) by regression method would be quite effective in correcting for the nonstationarity of the original series.[8] Other methods have also been discussed in the literature.[9]

Which method is the best? None is superior on a priori grounds. As Granger and Hatanaka note, "The leakage experiments (and connected theoretical work) seem to indicate that as long as *most* of the trend in mean is removed, then the spectral and cross-spectral methods may be used in confidence. As any of the methods considered effectively remove a very large part of the trend, it would seem that in most cases it is of little consequence which method one uses." They conclude that "the method to be used for removing trend, then, really depends on what subsequent analysis one intends to perform.... It is impossible to lay down any general rules for trend removal or estimation. The readers may use their own preferences."[10]

As for the use of the term "significant" in cross-spectral analysis, we agree with Dr. Percival that such use could be misleading. In our paper we note that there is no accepted theoretical method for determining what is a significantly high value of coherence. We decided, arbitrarily, that coherence values of 0.7 or above indicate that, at the particular frequency in question, the cross spectrum is significant. In doing so, we may have overlooked the fact that the sample cross spectrum has the same undesirable property as the sample spectrum, namely that its variance is independent of the record length and that this variance is quite large relative to the mean. Despite this difficulty, however, the sample cross spectrum may be used to construct a frequency domain test for cross correlation, between time series, based on the sample coherency and the sample cross spectrum.[11]

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[8] Granger and Hatanaka [1, p. 136] point out that regression techniques are particularly useful for data of moderate length. They add that "it is known both from theory and experience that polynomial regression and subtraction does not affect to any worthwhile extent the spectral estimates of frequency bands other than the band centered on zero frequency, which it effectively removes."

[9] See Jenkins and Watts [2].

[10] Granger and Hatanaka [1, p. 145]. Later on they explicitly recommend the use of regression trend removal method in many practical situations [1, p. 233].

[11] See Jenkins and Watts [2, Chapter 9] for a detailed description of the test.

Jenkins and Watts [2, pp. 357-368] indicate that a complete frequency domain description of a bivariate process requires a phase spectrum as well as a coherency spectrum. Thus, an indication of the correlation between two time series is provided not only by the coherency but also by the phase spectral estimator itself.[12] If the two processes are uncorrelated, the sample phase spectrum will be approximately uniformly distributed in the range $-\pi/2$, $\pi/2$. In such a case, the cumulative distribution function of the phase angle will be a straight line in this range. Our results, in the range of frequencies where the coherence level is quite high, fail the phase test for uncorrelated series which Jenkins and Watts suggest. This suggests to us that the two interest rate series are indeed correlated at these particular frequencies. The phase angle and the coherency spectrums of our results are given in Figure I for all the frequencies (including "nonsignificant" frequencies).

Finally, the level of significance is related to the variance of the phase spectrum. On this issue Jenkins and Watts say, "The results show that the variance (of the phase estimator) depends on the smoothing factor, which can be controlled by window closing, and on the coherency spectrum of the two processes. They also show that in all cases the variance of the estimator is zero when the coherency is unity and increases as the coherency tends to zero.... This is to be expected since low coherency implies a large noise level and hence an inefficient estimate."[13]

They go on to state that "the phase and coherency estimators are uncorrelated and hence it is permissible to derive confidence intervals for these spectra separately." As noted above, our coherence and phase estimates meet the tests that they outline in the range of frequencies which correspond to one up to two years' cycles.

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[12]The coherence itself plays the role of a correlation coefficient defined at each frequency, and values of squared coherency between 0 and 1 correspond to situations where $x_2(t)$ can be partially predicted from $x_1(t)$. The coherency spectrum in itself is useful in practice because it provides a measure of the correlation between two time series as a function of frequency. The cross correlation properties of two time series, however, should be described both by the squared coherence spectrum and by the phase spectrum.

[13]Jenkins and Watts [2, p. 379].

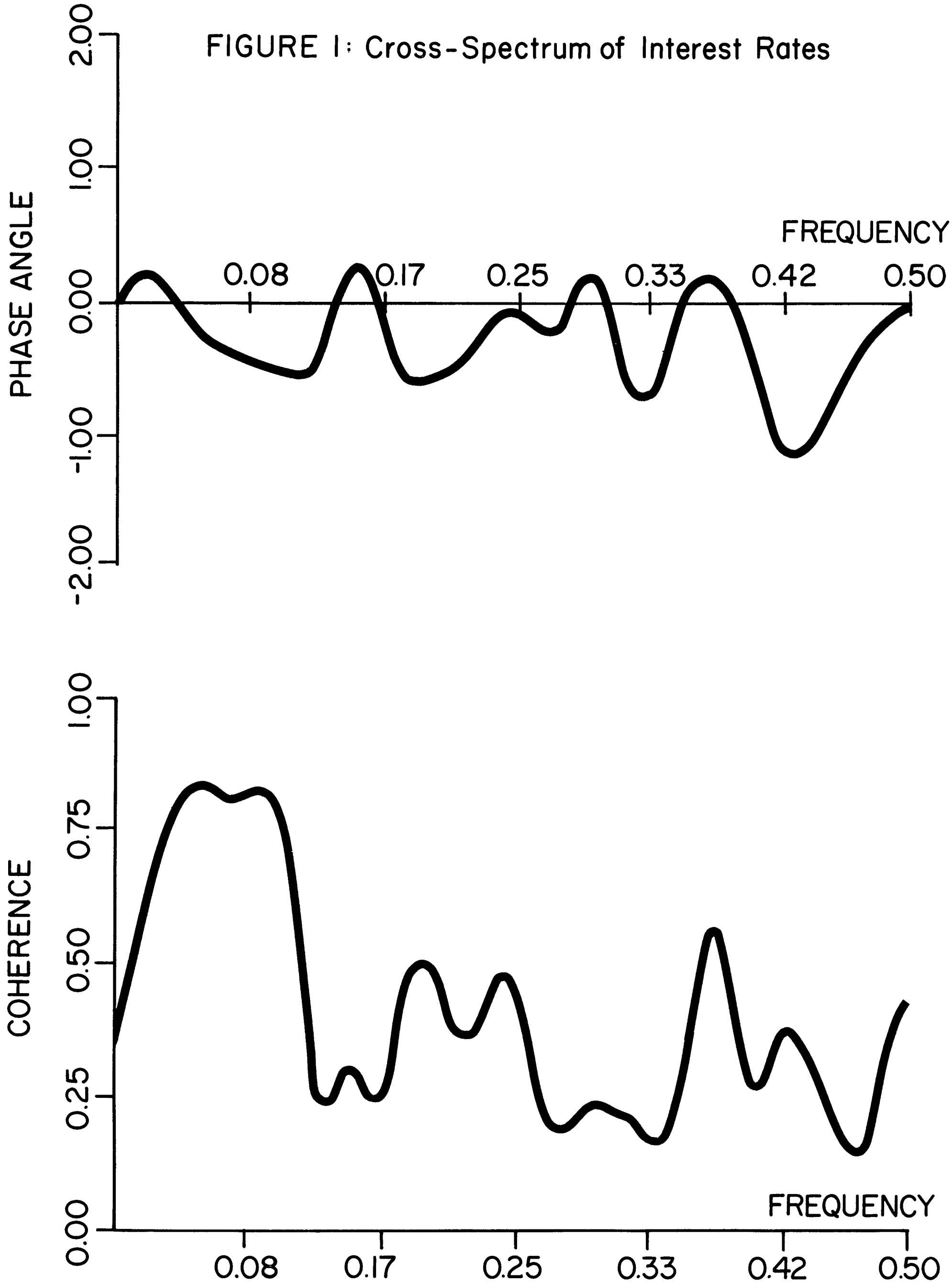
FIGURE 1: Cross-Spectrum of Interest Rates
PHASE ANGLE
2.00
1.00
0.00
-1.00
-2.00
FREQUENCY
0.08
0.17
0.25
0.33
0.42
0.50
COHERENCE
1.00
0.75
0.50
0.25
0.00
FREQUENCY
0.08
0.17
0.25
0.33
0.42
0.50

Therefore, conclude, as we did in our article, that a short-run cycle of eighteen to twenty-four months appears in both the ninety-day Treasury bill rate and the ten year U. S. government bond rate (as we defined them in [4]). Over the cycle, the long-term rate leads the short-term rate.

REFERENCES

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