Alternative Definitions of the Business Cycle and Their Implications for Business Cycle Models: A Reply to Torben Mark Pederson

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1 Introduction

In our 1995 *JEDC* paper, James Nason and I argued that the Hodrick-Prescott (HP) filter can produce "spurious" business cycles, in the sense that HP filtered data may exhibit business cycle dynamics even if none were present in the original data. Torben Mark Pederson (1998) criticizes our analysis on the grounds that we fail to define "business cycles" or what we mean by the term "spurious." He argues that if one defines business cycles in terms of an ideal high pass filter, then the HP filter cannot produce "spurious cycles," because it well approximates an ideal high pass filter. Indeed, based on this definition of the business cycle, our analysis would imply that even an ideal high pass filter generates a spurious cycle, a conclusion which is nonsensical.

Pederson's arguments are correct, provided that one accepts his definition of the business cycle. But Nason and I had a different definition in mind. I must plead guilty to the charge of failure to provide clear definitions, and in this note I try to clarify what we meant.

2 Two Definitions of the Business Cycle

Pederson adopts one common definition of business cycles, namely the component of a time series that passes through an ideal highpass (or bandpass) filter. For example, Hodrick and Prescott (1997) define the business cycle in terms of periodic components lasting 8 years or less, and Baxter and King (1995) define it in terms of components

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whose periodicities range from 2 to 8 years. Focusing on the HP definition, if y_t is the log level of the variable of interest, the HP cycle is

$$HP_t = F(L)y_t, \tag{1}$$

where

$$F(L) = \frac{\lambda(1-L)^2(1-L^{-1})^2}{1+\lambda(1-L)^2(1-L^{-1})^2}. \tag{2}$$

The parameter λ controls the smoothness of the trend component, and is typically set equal to 1600. This is a time-honored approach to measuring business cycles, and much has been learned from its application.

But there are other definitions. One prominent alternative is due to Beveridge and Nelson (1981), who define the business cycle in terms of predictable changes in the data:

$$BN_t = -E_t \sum_{j=1}^{\infty}[\Delta \ln y_{t+j} - \mu_y], \tag{3}$$

where $E_t$is the conditional expectations operator and μ_y is the unconditional mean of $\Delta \ln y_t$. The Beveridge-Nelson (BN) definition formalizes the intuition that expected growth in y_t should be higher than average when y_t is below its trend level. This is also a time-honored approach that has proven its value in applied work.

Although we were not clear on the point, Nason and I were interested in comparing these definitions. We were especially interested in whether the existence of business cycles in the HP sense implied the existence of business cycles in the BN sense. With a little reflection, it is easy to see that if there is a cycle in the BN sense, there will probably also be one in the HP sense, although there may be some compression of predictable long-horizon components. But a counterexample in our earlier paper demonstrated that the converse does not hold. If y_t is a random walk with drift, there is a cycle in the HP sense but not one in the BN sense. When we said that the HP filter may generate "spurious cycles," we meant that there could be HP cycles even if there were no BN cycles.

3 So What?

Why were we interested in that question? Its relevance is best illustrated by a brief (and highly selective) survey of the real business cycle literature. Following Kydland and Prescott (1982), most RBC modelers adopted the HP definition and focused on whether models could account for the periodicity of output, comovements between output and other variables, and the relative volatility of other variables to output. Here, I want to concentrate on the first dimension of the data, as this has been the focus of much of the recent literature.

Initially, it appeared that RBC models were extremely successful at replicating output dynamics. For example, Kydland and Prescott (1982) compared sample and model-generated autocorrelations of HP filtered output and found that they were very much alike. I think it is fair to say that many readers inferred that model propagation mechanisms (capital accumulation in general and time-to-build in particular) were quite powerful and that they could account for the business cycle periodicity (in the HP sense) found in U.S. data.

But eventually other researchers employing different methods came to different conclusions about the strength of model propagation mechanisms. For example, Rotemberg and Woodford (1996) compared sample and model-generated BN components and found that they were not at all alike. In U.S. data, there is a discernible BN cycle which is closely aligned with the dates of NBER recessions and expansions. In contrast, there is little predictable variation in model realizations. Rotemberg and Woodford concluded that first generation RBC models had weak propagation mechanisms and that they could not account for the predictable variation observed in the data.[1]

Why were the models successful in generating HP cycles, but unsuccessful in generating BN cycles? Roughly speaking, data on U.S. GDP are well approximated by the sum of a random walk and a stationary $AR(2)$ process with a hump-shaped moving average representation (e.g., see Rotemberg and Woodford). On the other hand, in early RBC models, output is well approximated by a random walk. The first representation generates cycles of both kinds; the latter generates an HP cycle but not a BN cycle. If the HP measure were the basis of comparison, one would conclude that first-generation RBC models well approximate the data. But if the BN measure were the basis of comparison, one would draw the opposite conclusion. This explains why researchers like Kydland and Prescott drew favorable conclusions about early RBC models, while researchers like Rotemberg and Woodford drew unfavorable ones.

In some sense, the problem is that it is too easy to generate HP cycles. In a conventional RBC model, one can do so just by assuming that technology shocks are persistent (which is certainly plausible). Modeling internal sources of propagation is unnecessary. But accounting for BN cycles is more demanding, because it requires better models of propagation. In the words of Hansen and Heckman (1996):

> "The models survive the weak standards for verification imposed by the calibrators. A much more disciplined and systematic exploration of the intertemporal and cross correlations, in a manner now routine in time series econometrics, would have shifted the focus from the empirical successes to the empirical challenges."

By highlighting shortcomings of first generation models, application of the Beveridge-Nelson measure (and other related measures) has helped shift the focus of the RBC literature toward the problem of modelling internal sources of propagation.

[1] Similar results were also reported by Christiano (1988), King, Plosser, and Rebelo (1988a and 1988b), Watson (1993), Cochrane (1994), and Cogley and Nason (1995a and 1995b).

4 Conclusion:

Pederson is correct to criticize us for failing to provide adequate definitions. The proper way to state our result is to say that the existence of business cycles in the sense of Hodrick and Prescott does not imply the existence of business cycles in the sense of Beveridge and Nelson. This distinction is relevant for business cycle analysis because first-generation RBC models generate HP cycles but not BN cycles. Empirical studies based on the latter definition were useful in diagnosing shortcomings of conventional RBC models and in reorienting research toward repairing those defects.

5 References

Beveridge, Stephen and Charles R. Nelson, 1981, "A New Approach to Decomposition of Economic Time Series Into Permanent and Transitory Components With Particular Attention to Measurement of the Business Cycle," *Journal of Monetary Economics* 7, pp. 151-174.

Christiano, Lawrence J., 1988, "Why Does Inventory Investment Fluctuate So Much?" *Journal of Monetary Economics* 21, 247-280.

Cochrane, John H., 1994, "Shocks," *Carnegie-Rochester Series on Public Policy* 41, pp. 295-364.

Cogley, Timothy and James M. Nason, 1995a, "Effects of the Hodrick-Prescott Filter on Trend and Difference Stationary Time Series: Implications for Business Cycle Research," *Journal of Economic Dynamics and Control* 19, pp. 253-278.

—— and —— , 1995b, "Output Dynamics in Real Business Cycle Models, *American Economic Review* 85, pp. 492-511.

Hodrick, Robert J. and Edward C. Prescott, 1997, "Post-War U.S. Business Cycles: An Empirical Investigation," Journal of Money Credit and Banking 29, pp. 1-16.

King, Robert G., Charles I. Plosser, and Sergio Rebelo, 1988a, "Production, Growth, and Business Cycles: The Basic Neoclassical Model," *Journal of Monetary Economics* 21, pp. 195-232.

—— , —— , and —— , 1988b, "Production, Growth, and Business Cycles: New Directions," *Journal of Monetary Economics* 21, pp. 309-342..

Kydland, Finn E. and Edward C. Prescott, 1982, "Time-to-Build and Aggregate Fluctuations," *Econometrica* 50, pp. 1345-1370.

Pederson, Torben Mark, 1998, "The Hodrick-Prescott Filter, the Slutzky Effect, and the Distortionary Effect of Filters," unpublished manuscript, Institute of Economics, University of Copenhagen.

Watson, Mark, 1993, "Measures of Fit for Calibrated Models," *Journal of Political Economy* 101, pp.119-132.